# Package ‘boussinesq’

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**License** GPL (>= 3)

**Title** Analytic Solutions for (Ground-Water) Boussinesq Equation

**Type** Package

**Depends** R (>= 2.10)

**Author** Emanuele Cordano

**Description** A collection of R functions were implemented from published and available analytic solutions for the One-Dimensional Boussinesq Equation (ground-water). In particular, the function ``beq.lin()'' is the analytic solution of the linearized form of Boussinesq Equation between two different head-based boundary (Dirichlet) conditions; ``beq.song'' is the non-linear power-series analytic solution of the motion of a wetting front over a dry bedrock (Song at al, 2007, see complete reference on function documentation). Bugs/comments/questions/collaboration of any kind are warmly welcomed.

**Version** 1.0.6

**Repository** CRAN

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**URL** https://github.com/ecor/boussinesq,https://agupubs.onlinelibrary.wiley.com/doi/10.1002/wrcr.20072

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`boussinesq-package` *Analytic solutions for (ground-water) Boussinesq Equation*

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## Description

Analytic solutions for (ground-water) Boussinesq Equation

## Author(s)

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`beq.lin` *Analytic exact solution for One-Dimensional Boussinesq Equation in a two-bounded domain with two constant-value Dirichlet Condition*

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## Description

Analytic exact solution for One-Dimensional Boussinesq Equation in a two-bounded domain with two constant-value Dirichlet Condition

## Usage

```
beq.lin(
  t = 0,
  x = seq(from = 0, to = L, by = by),
  h1 = 1,
  h2 = 1,
  L = 100,
  ks = 0.01,
  s = 0.4,
  big = 10^7,
  by = L/100,
  p = 0.5
)
```

### Arguments

| | |
|---|---|
| `t` | time coordinate. |
| `x` | spatial coordinate. Default is `seq(from=0,to=L,by=by)`. |
| `h1` | water surface level at `x=0`. Left Dirichlet Bounday Condition. |
| `h2` | water surface level at `x=L`. Right Dirichlet Bondary Condition. |
| `L` | length of the domain. |
| `ks` | Hydraulic conductivity |
| `s` | drainable pororosity (assumed to be constant) |
| `big` | maximum level of Fourier series considered. Default is 10^7. |
| `by` | see `seq` |
| `p` | empirical coefficient to estimate hydraulic diffusivity $D = ks/(s*(p*h1+(1-p)*h2))$. It ranges between 0 and 1. |

### Value

Solutions for the indicated values of `x` and `t`.

### Author(s)

Emanuele Cordano

### See Also

`beq.lin.dimensionless`

### Examples

```
L <- 1000
x <- seq(from=0,to=L,by=L/100)
t <- 4 # 4 days
h_sol0 <- beq.lin(x=x,t=t*24*3600,h1=2,h2=1,ks=0.01,L=L,s=0.4,big=100,p=0.0)
h_solp <- beq.lin(x=x,t=t*24*3600,h1=2,h2=1,ks=0.01,L=L,s=0.4,big=100,p=0.5)
h_sol1 <- beq.lin(x=x,t=t*24*3600,h1=2,h2=1,ks=0.01,L=L,s=0.4,big=100,p=1.0)

plot(x,h_sol0,type="l",lty=1,main=paste("Water Surface Elevetion after",
t,"days",sep=" "),xlab="x[m]",ylab="h[m]")
lines(x,h_solp,lty=2)
lines(x,h_sol1,lty=3)
legend("topright",lty=1:3,legend=c("p=0","p=0.5","p=1"))
```

---

`beq.lin.dimensionless` *Analytic exact solution for Dimentionless (i. e. diffusivity equal to 1 - unity) One Dimensional Heat Equation in a two-bounded domain with two constant-value Dirichlet Conditions*

---

## Description

Analytic exact solution for Dimentionless (i. e. diffusivity equal to 1 - unity) One Dimensional Heat Equation in a two-bounded domain with two constant-value Dirichlet Conditions

## Usage

```
beq.lin.dimensionless(
  t = 0,
  x = seq(from = 0, to = L, by = by),
  big = 1e+05,
  by = L * 0.01,
  L = 1
)
```

## Arguments

| | |
|---|---|
| `t` | time coordinate. |
| `x` | spatial coordinate. Default is `seq(from=0,to=L,by=by)`. |
| `big` | maximum level of Fourier series considered. Default is 100000. |
| `by` | see `seq` |
| `L` | length of the domain. It is used if `x` is not specified. |

## Value

Solutions for the specifiied values of `x` and `t`

## Author(s)

Emanuele Cordano

## References

Rozier-Cannon, J. (1984), The One-Dimensional Heat Equation, Addison-Wesley Publishing Company, Manlo Park, California, encyclopedia of Mathematics and its applications.

## See Also

`beq.lin`

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`beq.song` *Song et al.'s analytic solution to Boussinesq equation in a 1D semi-infinite domain with a Dirichlet boundary condition*

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### Description

Song et al.'s analytic solution to Boussinesq equation in a 1D semi-infinite domain with a Dirichlet boundary condition

### Usage

```
beq.song(t = 0.5, x = 1, s = 0.4, h1 = 1, ks = 0.01, nmax = 4, alpha = 1)
```

### Arguments

| | |
|---|---|
| `t` | time coordinate. |
| `x` | spatial coordinate. Default is `seq(from=0,to=L,by=by)`. |
| `s` | drainable pororosity (assumed to be constant) |
| `h1` | water surface level or boundary condition coefficient at `x=0`. Left Dirichlet Bounday Condition. |
| `ks` | Hydraulic conductivity |
| `nmax` | order of power series considered for the analytic solution solution. Default is 4. |
| `alpha` | $\alpha$ exponent see Song at al, 2007 |

### Value

The water surface eletion vs time and space obtained by the analytic solution of Boussinesq Equation

### Note

For major details, see Song at al, 2007

### Author(s)

Emanuele Cordano

### References

Song, Zhi-yao;Li, Ling;David, Lockington. (2007), "Note on Barenblatt power series solution to Boussinesq equation",Applied Mathematics and Mechanics, https://link.springer.com/article/10.1007/s10483-007-0612-x ,doi:10.1007/s104830070612x

### See Also

`beq.song.dimensionless`

### Examples

```
L <- 1000
x <- seq(from=0,to=L,by=L/100)
t <- c(4,5,20) #  days

h_sol1 <- beq.song(t=t[1]*3600*24,x=x,s=0.4,h1=1,ks=0.01,nmax=10,alpha=0)
h_sol2 <- beq.song(t=t[2]*3600*24,x=x,s=0.4,h1=1,ks=0.01,nmax=10,alpha=0)
h_sol3 <- beq.song(t=t[3]*3600*24,x=x,s=0.4,h1=1,ks=0.01,nmax=10,alpha=0)


plot(x,h_sol1,type="l",lty=1,
main="Water Surface Elevetion (Song at's solution) ",
xlab="x[m]",ylab="h[m]")
lines(x,h_sol2,lty=2)
lines(x,h_sol3,lty=3)
legend("topright",lty=1:3,legend=paste("t=",t,"days",sep=" "))
```

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## beq.song.dimensionless

*Dimensionless solution for one-dimensional derived equation from scaling Boussinesq Equation (Song et al, 2007)*

---

### Description

Dimensionless solution for one-dimensional derived equation from scaling Boussinesq Equation (Song et al, 2007)

### Usage

```
beq.song.dimensionless(xi, xi0, a)
```

### Arguments

| | |
|---|---|
| `xi` | dimensionless coordinate (see `Note`) |
| `xi0` | displacement of wetting front expressed as dimensionless coordinate (see `Note`) |
| `a` | vector of coefficient returned by `coefficient.song.solution` |

### Value

the dimesioneless solution, i.e. the variable $H$

### Note

The expession for the dimensionless coordinate (Song at al., 2007) is $\xi = x(\frac{2\,s}{\eta_1\,K_s\,t^{\alpha+1}})^{1/2}$ and the solution for the dimensionless equation derived by Boussinesq Equation is: $H = \sum_{n=0}^{\infty} a_n (1-\frac{\xi}{\xi_0})^n$ for $\xi < \xi_0$ , otherwise is 0 .

### Author(s)

Emanuele Cordano

### References

Song, Zhi-yao;Li, Ling;David, Lockington. (2007), "Note on Barenblatt power series solution to Boussinesq equation",Applied Mathematics and Mechanics, https://link.springer.com/article/10.1007/s10483-007-0612-x ,doi:10.1007/s104830070612x

### See Also

beq.song

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coefficient.song.solution

*Alogorithm for resolution of the series coefficient a_n for the dimensionless formula for $H$ in* beq.song.dimensionless

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### Description

Alogorithm for resolution of the series coefficient $a_n$ for the dimensionless formula for $H$ in beq.song.dimensionless

### Usage

```
coefficient.song.solution(n = 4, lambda = 0)
```

### Arguments

| | |
|---|---|
| n | approximation order |
| lambda | dimensionless parameter related to $\alpha$ see Song at al, 2007 |

### Value

the $a_n$ series coefficient

### Note

For major details, see Song at al, 2007

### Author(s)

Emanuele Cordano

### References

Song, Zhi-yao;Li, Ling;David, Lockington. (2007), "Note on Barenblatt power series solution to Boussinesq equation",Applied Mathematics and Mechanics, https://link.springer.com/article/10.1007/s10483-007-0612-x ,doi:10.1007/s104830070612x

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